

Module Examination

Essential mathematics 2

Time allowed: 3 hours

This paper has THREE sections. You should attempt ALL questions in Sections A and B, and TWO questions from Section C.

Section A carries 36% of the marks. It has 18 computer-marked questions, each worth 2%.

Section B carries 40% of the marks. It has 5 questions, each worth 8%.

Section C carries 24% of the marks. It has 3 questions, each worth 12%, of which you should attempt two.

For **Section A**, mark your answers on the computer marked examination (CME) form provided, using an HB pencil. Instructions for filling in the CME form are given overleaf. The assignment number for this examination is **MST125 81**. Write your rough work in the answer book(s) provided. It will not be considered by the examiners.

For **Sections B** and **C**, write your answers in **pen** in the answer book(s) provided. Start your answer to each question on a new page. Include all your working. Do not cross out any work until you have written a new attempt.

At the end of the examination

- Make sure that you have completed Part 1 of the CME form, and written your personal identifier and your examination number on each answer book used. **Failure to do so will mean that your work cannot be identified.**
- Make sure that you have entered one answer (A, B, C, D or E) against each of questions 1–18 in Part 2 of the CME form.
- Attach your signed desk record to the front of your answer book(s) using the round paper fastener, then attach the CME form and your question paper to the back of the answer book(s) using the flat paperclip.
It is important that the flat paper clip is used with the CME form since the computer cannot read CME forms with punched holes.

Instructions for filling in the computer-marked examination (CME) form

If you do not follow these instructions, then the examiners may not be able to award you a score for Section A of the examination.

You will find one CME form provided with this paper. The invigilator has a supply of spare forms.

Writing on the form

- Use an HB pencil.
- To mark a cell, pencil across it, as demonstrated on the form.
- To cancel a mark, pencil in the coloured part of the cell, as demonstrated on the form.
- If you make any unwanted marks on the form that you cannot cancel clearly, then ask the invigilator for a new form, and transfer your entries to it.

Completing Part 1

- Write your personal identifier (NOT your examination number) and the assignment number for this examination, which is **MST125 81**, in the boxes provided.
- In the blocks headed 'PERSONAL IDENTIFIER' and 'MODULE AND ASSIGNMENT NUMBER', pencil across the cells corresponding to your personal identifier and the assignment number given above.
- On the sample CME form opposite, Part 1 has been completed for a fictitious student so that you can see how to complete that part of the form.

Completing Part 2

- For each question (numbered 1 to 18) in Section A of the examination paper, mark your answer by pencilling across ONE of cells A, B, C, D or E.
- If you think that a question is unsound in any way, pencil across the 'unsound' cell (U), *as well as* pencilling across an answer cell.
- Do not pencil across any other cell.

COMPUTER MARKED EXAMINATION FORM

08-10

PART 1

PERSONAL IDENTIFIER
(NOT Examination Number)

A7654321

ASSIGNMENT NUMBER
(as given on the question paper)

MST125 81

NAME and INITIALS

A. STUDENT

IMPORTANT NOTES

1. Please check that all of this form has been completed including the coloured boxes in Part 1 with your Personal Identifier, Module and Assignment Number.
2. Only use an HB pencil to complete this form and press firmly.
3. Shown opposite are examples of the correct marks to use. Do not use any other types of mark.

**PLEASE DO NOT PUNCH HOLES
IN THIS FORM**

Fold this form INWARDS along the dotted line

PERSONAL IDENTIFIER

A	0	0	0	0	0	0	0
B	1	1	1	1	1	1	1
C	2	2	2	2	2	2	2
D	3	3	3	3	3	3	3
E	4	4	4	4	4	4	4
F	5	5	5	5	5	5	5
G	6	6	6	6	6	6	6
H	7	7	7	7	7	7	7
K	8	8	8	8	8	8	8
L	9	9	9	9	9	9	9
M							
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X							
Y							

CORRECT MARK

Pencil across
between dots



CANCELLATION MARK

To cancel a mark
pencil in the
coloured part of
the cell. Do not rub out



MODULE AND ASSIGNMENT NUMBER

A	A	A	A	0	0	0	0
B	B	B	B	1	1	1	1
C	C	C	C	2	2	2	2
D	D	D	D	3	3	3	3
E	E	E	E	4	4	4	4
F	F	F	F	5	5	5	5
G	G	G	G	6	6	6	6
H	H	H	H	7	7	7	7
K	K	K	K	8	8	8	8
L	L	L	L	9	9	9	9
M	M	M	M				
N	N	N	N				
P	P	P	P				
R	R	R	R				
S	S	S	S				
T	T	T	T				
U	U	U	U				
W	W	W	W				
X	X	X	X				
Y	Y	Y	Y				
Z	Z	Z	Z				

OFFICE USE ONLY

0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

OFFICE USE ONLY



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PART 2

ANSWER

ANSWER

ANSWER

1	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
2	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
3	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
4	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
5	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
6	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
7	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
8	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
9	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
10	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
11	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
12	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
13	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
14	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
15	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
16	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
17	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
18	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
19	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
20	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
21	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
22	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
23	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
24	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
25	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
26	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
27	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
28	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
29	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
30	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
31	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
32	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
33	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
34	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
35	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
36	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
37	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
38	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
39	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
40	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
41	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
42	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
43	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
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45	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
46	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
47	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
48	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
49	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
50	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
51	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
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59	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
60	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U

SECTION A

Attempt **ALL** questions in this section. Each question is worth 2%.
Mark your answers in **pencil** on the CME form.

Question 1

What is the least residue of 6^{24} modulo 23 ?

- A** 1 **B** 6 **C** 9 **D** 13 **E** 17

Question 2

A parabola in standard position has equation $4y^2 - 3x = 0$. What is the equation of its directrix?

- A** $x = -3$ **B** $x = -\frac{3}{4}$ **C** $x = -\frac{3}{8}$ **D** $x = -\frac{3}{16}$ **E** $x = \frac{3}{4}$

Question 3

What is the eccentricity of the ellipse with equation

$$\frac{x^2}{16} + \frac{y^2}{2} = 1 ?$$

- A** $\frac{1}{8}$ **B** $\frac{1}{2\sqrt{2}}$ **C** $\frac{\sqrt{7}}{2\sqrt{2}}$ **D** $\frac{3}{2\sqrt{2}}$ **E** $\frac{7}{8}$

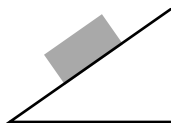
Question 4

Which of the following parametrisations represents the line segment that joins the points $(-1, 2)$ and $(2, 4)$?

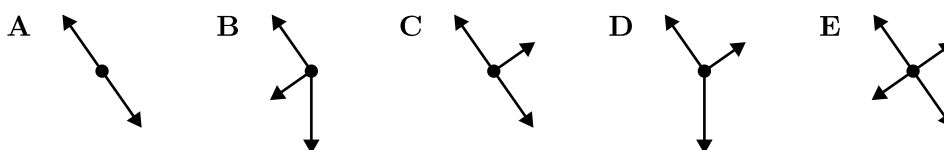
- A** $x = -1 + 3t, \quad y = 2 + 2t \quad (0 \leq t \leq 1)$
B $x = -1 + 3t, \quad y = 2 + 2t \quad (-1 \leq t \leq 2)$
C $x = 2 + 2t, \quad y = 4 - 2t \quad (0 \leq t \leq 2)$
D $x = 1 + 2t, \quad y = 2 + 4t \quad (0 \leq t \leq 1)$
E $x = 1 + 2t, \quad y = 2 + 4t \quad (-1 \leq t \leq 2)$

Question 5

A block rests on an inclined flat surface, as shown. The block is prevented from sliding down the slope by the friction between the block and the surface.

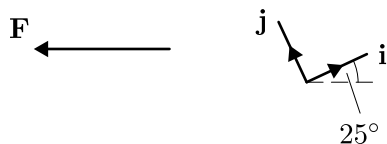


Which of the following force diagrams most accurately represents the main forces acting on the block?



Question 6

The force $\mathbf{F}$ acts horizontally to the left and has magnitude 8.3 N. The Cartesian unit vectors $\mathbf{i}$ and $\mathbf{j}$ have the directions shown, with $\mathbf{i}$ at an angle of 25° to the horizontal.



What is the component form of $\mathbf{F}$, in newtons to two significant figures?

- A $-8.0\mathbf{i} + 2.1\mathbf{j}$ B $-7.5\mathbf{i} + 3.5\mathbf{j}$ C $7.5\mathbf{i} - 3.5\mathbf{j}$
D $7.5\mathbf{i} + 3.5\mathbf{j}$ E $8.0\mathbf{i} + 2.1\mathbf{j}$

Question 7

Which of the following options describes the linear transformation $f(x, y) = (-y, -x)$?

- A Reflection in the line $y = x$
B Reflection in the line $y = -x$
C Rotation through π about the origin
D Rotation through $\pi/2$ about the origin
E Rotation through $-\pi/2$ about the origin

Question 8

Let $f(\mathbf{x}) = \mathbf{P}\mathbf{x}$ and $g(\mathbf{x}) = \mathbf{Q}\mathbf{x}$, where

$$\mathbf{P} = \begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix} \quad \text{and} \quad \mathbf{Q} = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}.$$

Which of the following matrices represents the composite transformation $g \circ f$?

A $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ **B** $\begin{pmatrix} 3 & 4 \\ 1 & -4 \end{pmatrix}$ **C** $\begin{pmatrix} 4 & 4 \\ 1 & -4 \end{pmatrix}$ **D** $\begin{pmatrix} 5 & 4 \\ 5 & -2 \end{pmatrix}$
E $\begin{pmatrix} 5 & 10 \\ 2 & -2 \end{pmatrix}$

Question 9

What is the form of the partial fraction expansion of the expression

$$\frac{4x^2 + 9x + 12}{(x-3)(x+2)^2} ?$$

In the options, A , B and C represent non-zero constants.

A $\frac{A}{x-3} + \frac{B}{x+2} + \frac{C}{x^2+2}$ **B** $\frac{A}{x-3} + \frac{B}{(x+2)^2}$
C $\frac{A}{x-3} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$ **D** $\frac{A}{x-3} + \frac{B}{x+2}$
E $\frac{A}{(x-3)(x+2)} + \frac{B}{(x+2)^2}$

Question 10

What is the quotient on dividing the polynomial expression $2x^2 + 5x - 3$ by the polynomial expression $x + 4$?

A -15 **B** 9 **C** $x + \frac{3}{2}$ **D** $2x - 3$ **E** $2x + 3$

Question 11

What is the general solution of the differential equation

$$\frac{dy}{dx} = y \left(\frac{1}{x} + 1 \right) \quad (x > 0, y > 0) ?$$

In the options, c is an arbitrary constant and A is an arbitrary positive constant.

A $y = \ln x + x + c$ **B** $y = Axe^x$ **C** $y = A \ln x + c$
D $y = \frac{Ae^x}{x}$ **E** $y = x \ln x + c$

Question 12

Which of the following is an integrating factor $p(x)$ for the differential equation

$$\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x} \quad (x > 0) ?$$

- A** $p(x) = \ln x$ **B** $p(x) = e^{x^2}$ **C** $p(x) = e^x$
D $p(x) = x$ **E** $p(x) = x^2$

Question 13

Let P be the following statement:

If n is a multiple of 6, then n is even.

Which of the following statements is the contrapositive of P ?

- A** If n is even, then n is a multiple of 6.
B If n is odd, then n is a multiple of 6.
C If n is odd, then n is not a multiple of 6.
D If n is not a multiple of 6, then n is even.
E If n is not a multiple of 6, then n is odd.

Question 14

An object moving along a straight line slows down at the rate of 3 m s^{-2} and comes to a halt after 12 seconds. What is the distance travelled by the object during this time?

- A** 18 m **B** 24 m **C** 36 m **D** 204 m **E** 216 m

Question 15

Which of the following numbers is an eigenvalue of the matrix $\begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix}$ corresponding to the eigenvector $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$?

- A** -1 **B** 1 **C** 2 **D** 3 **E** 4

Question 16

What is the probability of getting two numbers each greater than 4 when two dice are rolled?

- A** $\frac{1}{9}$ **B** $\frac{1}{3}$ **C** $\frac{4}{9}$ **D** $\frac{1}{2}$ **E** $\frac{2}{3}$

Question 17

How many positive integers less than 500 have at least one 6 in their decimal representation?

- A** 94 **B** 95 **C** 96 **D** 175 **E** 176

Question 18

What is the general solution of the recurrence relation

$$u_n = -\frac{3}{2}u_{n-1} + u_{n-2} \quad (n = 2, 3, 4, \dots)?$$

- A** $A \times (-2)^n + B \times \left(\frac{1}{2}\right)^n$ **B** $A \times \left(\frac{1}{2}\right)^n + B \times 2^n$
C $A \times \left(\frac{3}{2}\right)^n + B \times \left(\frac{1}{2}\right)^n$ **D** $A \times \left(-\frac{3}{2}\right)^n + B$
E $(A + Bn) \times \left(-\frac{3}{2}\right)^n$

TURN OVER FOR SECTION B

SECTION B

Attempt **ALL** questions in this section. Each question is worth 8%.

Write in **pen**. **Include all your working**, as some marks are awarded for this.

Start your answer to each question on a new page of your answer book.

Question 19

- (a) Use Euclid's algorithm to find the highest common factor d of 87 and 38. [3]
- (b) Hence find two integers v and w such that $87v + 38w = d$. State the values of v and w . [5]

Question 20

Using the substitution $x = 3 \cosh u$, or otherwise, find the integral

$$\int \frac{x^2}{\sqrt{x^2 - 9}} dx. \quad [8]$$

Question 21

Let $P(n)$ be the following statement:

$$(n + 1)! > 3^n.$$

- (a) Determine whether $P(n)$ is true or false for each of $n = 3$ and $n = 4$. [2]
- (b) Use mathematical induction to show that $P(n)$ is true for all $n \geq 4$. [6]

Question 22

A rugby ball is kicked into the air from the ground. When it leaves the ground, it is travelling at a speed of 10 m s^{-1} at an angle of 60° to the horizontal.

Take the x -axis to point horizontally in the direction of motion, and the y -axis to point vertically upwards, with the origin at the starting point of the ball. Assume that there are no forces from the air on the ball.

- (a) Find the velocity of the ball when it leaves the ground, in component form. [2]
- (b) Find an expression in component form for the position $\mathbf{r}$ (in metres) of the ball in terms of the time t (in seconds) since it left the ground. Denote the magnitude of the acceleration due to gravity by g . [6]

Question 23

Express the matrix $\begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix}$ in the form $\mathbf{P}\mathbf{D}\mathbf{P}^{-1}$, where $\mathbf{D}$ and $\mathbf{P}$ are 2×2 matrices, and $\mathbf{D}$ is a diagonal matrix. Evaluate $\mathbf{P}^{-1}$ as part of your answer.

[8]

TURN OVER FOR SECTION C

SECTION C

Attempt **TWO** questions from this section. Each question is worth 12%.

Write in **pen**. **Include all your working**, as some marks are awarded for this.

Start your answer to each question on a new page of your answer book.

If you answer three questions, you will be awarded the marks for your best two answers only.

Question 24

Let f be the linear transformation that maps the point $(1, 0)$ to the point $(2, 8)$, and the point $(0, 1)$ to the point $(1, 3)$.

(a) Write down the matrix that represents f . [1]

(b) Show that f is invertible, and find the matrix that represents f^{-1} . [2]

(c) Find the equation of the image $f(\mathcal{C})$ of the unit circle $\mathcal{C}$, in the form

$$ax^2 + bxy + cy^2 = d,$$

where a, b, c and d are integers whose values you should find. [5]

(d) Check the equation that you found in part (c) by substituting into it the coordinates of a known point on $f(\mathcal{C})$. [2]

(e) Let g be the translation by 1 unit to the right and 2 units down. Find the rule of the affine transformation $f \circ g$ in the form

$$(f \circ g)(\mathbf{x}) = \mathbf{Ax} + \mathbf{a},$$

where $\mathbf{A}$ is a 2×2 matrix and $\mathbf{a}$ is a 2-dimensional vector. [2]

Question 25

Consider the function $f(x) = \frac{x}{x^2 - 1}$.

(a) Find its domain and intercepts. [2]

(b) Find $f'(x)$. [2]

(c) Construct a table of signs for $f'(x)$, determine the intervals on which f is increasing or decreasing and find any stationary points and their natures. [3]

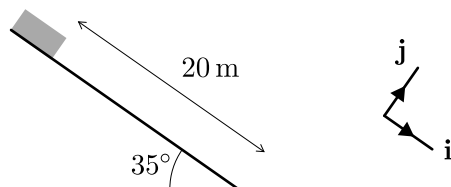
(d) Find the asymptotes of f . [2]

(e) Determine whether f is an even or odd function, or neither. [1]

(f) Sketch the graph of f . [2]

Question 26

A box is released from rest at the top of a flat rough slope of length 20 metres that is inclined at an angle of 35° to the horizontal. It slides under gravity to the bottom of the slope. The coefficient of sliding friction between the box and the slope is 0.17. Take the magnitude of the acceleration due to gravity to be $g = 9.8 \text{ ms}^{-2}$. Model the box as a particle.



- (a) State the three forces acting on the box during its motion, and draw a force diagram representing these forces, labelling them clearly. [3]
- (b) Find expressions for the component forms of the three forces, in terms of the mass m (in kg) of the box and any unknown magnitude(s) where appropriate. In doing this, take the Cartesian unit vectors $\mathbf{i}$ and $\mathbf{j}$ to point parallel and perpendicular to the slope in the directions shown above. [2]
- (c) Hence or otherwise find the magnitude of the acceleration of the box, in ms^{-2} to two significant figures. [5]
- (d) Find the time that the box takes to reach the bottom of the slope, in seconds to two significant figures. [2]

[END OF QUESTION PAPER]